Correction to "Intersection Cuts with Infinite Split Rank"

Amitabh Basu[1], Gérard Cornuéjols[2,3,4]
François Margot[2,5]

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Abstract

We point out that the statement of Lemma 2.2 (ii) in [1] is incorrect. We fix the statement and show that this error does not impact the other results of the paper.

1 Correction to Lemma 2.2

The following is a corected version of Lemma 2.2 in [1]. The correction is in bold.

Lemma 2.2. *Let Q be the linear relaxation of $X = \{(x, y) \mid Ax + By \geq b,\ x \in \mathbb{Z}^p, y \in \mathbb{R}^q\}$.*

(ii) Let y' be a subset of the y variables and let $Q(x, y')$ be the orthogonal projection of Q onto the variables (x, y'). Consider a valid inequality $\mathcal{I}$ for conv(X) whose coefficients for the y variables not in y' are all 0. The split rank of inequality $\mathcal{I}$ with respect to $Q(x, y')$ is **greater than or equal** *to its split rank with respect to Q.*

The original statement had "identical" instead of "greater than or equal". The proof of the modified statement is below (modification in bold).

Proof. (ii) Let proj be the operation of projecting orthogonally onto the variables (x, y'). It follows from the definitions of projection and convex hull that the operations of taking the projection and taking the convex hull commute. Therefore we have, for any split (π, π_0) on the x variables,

$$\text{proj}\left(\text{conv}(Q^{\leq} \cup Q^{\geq})\right) = \text{conv}\left(\text{proj}(Q^{\leq}) \cup \text{proj}(Q^{\geq})\right) = \text{conv}\left(Q(x, y')^{\leq} \cup Q(x, y')^{\geq}\right) ,$$

with the validity of the last equality coming from the fact that none of the variables involved in the disjunction are projected out. For all $t = 0, 1, 2, \ldots$, the projection proj of the rank-t split closure of Q **is contained** in the rank-t split closure of $Q(x, y')$, **as the projection of an intersection of polyhedra is contained in the intersection of their projections**. The result then follows from the fact that inequality $\mathcal{I}$ is valid for a polyhedron Q' in the (x, y)-space if and only if it is valid for proj(Q'). □

[1] Dept. of Mathematics, University of California, Davis, CA 95616.

[2] Tepper School of Business, Carnegie Mellon University, Pittsburgh, PA 15213.

[3] LIF, Faculté des Sciences de Luminy, Université de Marseille, France.

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Use of point Lemma 2.2 (ii) can either be replaced or modified without altering any of the main results of the paper. Modifications are listed below.

1. Page 6, below equation (3): (modifications in bold)

   Clearly, P is the orthogonal projection of $P^L(x, s, z)$ onto the (x, s)-space. **As the relation between P and $P^L(x, s, z)$ is a bijection projecting or adding a single continuous variable** z, the split rank of (2) with respect to P or $P^L(x, s, z)$ are identical. Let $P^L(x, z)$ be the orthogonal projection of $P^L(x, s, z)$ onto the (x, z)-space. By Lemma 2.2 (iii), inequalities (2) and $z \leq 0$ have the same split rank with respect to $P^L(x, s, z)$. By Lemma 2.2 (ii), the split rank of the inequality $z \leq 0$ for $P^L(x, s, z)$ **is smaller than or equal to its rank for** $P^L(x, z)$. We thus have the following:

   Observation 3.1. *Let $L \subseteq \mathbb{R}^m$ be a lattice-free polytope containing f in its interior and let P be the linear relaxation of (1). The split rank of the L-cut (2) with respect to P is* **smaller than or** *equal to the split rank of the inequality $z \leq 0$ with respect to $P^L(x, z)$.*

2. Page 7: The last sentence of Observation 3.2 should be removed to get:

   Observation 3.2. *Let $L \subseteq \mathbb{R}^m$ be a lattice-free polytope containing f in its interior. If the rank-t split closure of P^L has height at most zero, then the split rank of the L-cut (2) with respect to P is at most t.*

3. Page 7: Proof of Theorem 4.1: Remove the reference to Observation 3.2 and do the proof for $P^L(x, s, z)$ instead of $P^L(x, z)$.

References

[1] A. Basu, G. Cornuéjols, F. Margot, "Intersection Cuts with Infinite Split Rank", *Mathematics of Operations Research* 37 (2012), 21–40